[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2594 **GC-3** **Your Roll No................**

Unique Paper Code : 32355301

Name of the Paper : Differential Equations

Name of the Course : **Generic Elective – 3 for Hons. Courses, Under CBCS**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.

2. Attempt **all** questions by selecting any **two** parts from each question.

1. (a) Find an integrating factor and solve the differential equation :

$$\left(e^{x+y} - y\right)dx + \left(xe^{x+y} + 1\right)dy = 0. \qquad (6.5)$$

(b) Solve the equation : $y' + (x + 1)y = e^{x^2} y^3$, $y(0) = 0.5$. (6.5)

(c) Find the orthogonal trajectories of the family of parabolas $y = ce^{-3x}$. (6.5)

2. (a) Show that e^{3x} and xe^{3x} form a basis of the following differential equation $y'' - 6y + 9y - 0$. Find also the solution that satisfies the conditions $y(0) = -1.4$, $y'(0) = 4.6$. (6)

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(b) Solve the initial value problem : (6)

$$x^2y'' + 3xy' + y = 0, \; y(1) = 4, \; y'(1) = -2.$$

(c) Find the radius of convergence of the series : $\sum_{m=2}^{\infty} \frac{(-1)^m (x-1)^{2m}}{4^m}$. (6)

3\. (a) Find a general solution of the following nonhomogeneous differential equation :

$$y'' + 3y' + 2y = 30e^{2x}$$

using variation of parameters. (6.5)

(b) Use the method of undetermined coefficients to find the particular solution of the differential equation : $y'' - 4y' + 4y = 2e^{2x}$. (6.5)

(c) Find a homogeneous linear ordinary differential equation for which two functions x^3 and x^{-2} are solutions. Show also linear independence by considering their Wronskian. (6.5)

4\. (a) Find the general solution of the partial differential equation

$$yzu_x - xzu_y + xy(x^2 + y^2)u_z = 0.$$ (6)

(b) Find a general solution of the differential equation :

$$(x^2D^2 + xD - 4I)y = 0, \text{ where } D = \frac{d}{dx}.$$ (6)

(c) Find the particular solution of the linear system that satisfies the stated initial conditions :

$$\frac{dy_1}{dt} = -5y_1 + 2y_2, \; y_1(0) = 1$$

$$\frac{dy_2}{dt} = 2y_1 - 2y_2, \; y_2(0) = -2$$ (6)

5. (a) Find a power series solution of the following differential equation, in powers of x

$$y'' + xy' - 2y = 0.$$ (6.5)

(b) Find the solution of the quasi-linear partial differential equation :

$$u(x + y)u_x + u[x - y)u_y = x^2 + y^2$$

with the Cauchy data $u = 0$ on $y = 2x$. (6.5)

(c) Reduce the equation : $yu_x + u_y = x$

to canonical form, and obtain the general solution. (6.5)

6. (a) Solve the initial-value problem :

$$u_x + 2u_y = 0, \quad \mu(0, y) = 4e^{-2y}$$

using the method of separation of variables. (6)

(b) Obtain the canonical form of the equation : $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$, and hence find the general solution. (6)

(c) Reduce the following partial differential equation with constant coefficients,

$$u_{xx} + 2u_{xy} + 5u_{yy} + u_x = 0.$$

into canonical form. (6)